

# Thermo-economic multi-objective optimization of solar dish-Stirling engine by implementing evolutionary algorithm



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## ABSTRACT

In the recent years, remarkable attention is drawn to Stirling engine due to noticeable advantages, for instance a lot of resources such as biomass, fossil fuels and solar energy can be applied as heat source. Great number of studies are conducted on Stirling engine and finite time thermo-economic is one of them. In the present study, the dimensionless thermo-economic objective function, thermal efficiency and dimensionless power output are optimized for a dish-Stirling system using finite time thermo-economic analysis and NSGA-II algorithm. Optimized answers are chosen from the results using three decision-making methods. Error analysis is done to find out the error through investigation.

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## 1. Introduction

At this moment it is very well known that the Finite Time Thermodynamics has been criticized because did not take into account the internal irreversibilities in the study of the real cycles [1–3]. Because of that any optimization based on it would not get results very closed to the reality of Thermal Machines. In order to improve the situation Ibrahim et al. [4] introduced for the first time a “coefficient of irreversibility” which would take into account the internal irreversibilities. This was done similar with the one introduced by Chen in [5] (in 2012) and implemented in their formulas but much earlier, in 1991. Unfortunately despite of the fact that this idea “improved the situation” of Thermodynamics with Finite Speed this could not get the Validation, drawing parallel with the computation with experimental results. Only the progresses made in the last 20 years in the field of Thermodynamics with Finite Speed [6–12] made it possible to get a Validation for 12 Stirling Machines [9] and 16 regimes of functioning. Based on the previous literature surveys [6–8] a very important Validation was obtained for Solar Stirling Engines [10–12].

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Performance analysis of heat engines applying finite-time thermodynamics were begun by Chambadal [13] and Novikov [14] and were followed by Moran [3]. In the first place, the performance of an endoreversible Carnot heat engine at maximum power output was scrutinized by Curzon and Ahlborn [15]. Many optimization researches on heat engines based on endoreversible and irreversible models have been done with respect to finite-time and finite-size constraints under various heat transfer modes, largely linear and non-linear ones, over the decades. Some optimization works could be found in literature survey represented by Bejan [16], Chen et al. [17] and Durmayaz et al. [18]. Typically, the optimization criteria are; the power output, thermal efficiency, entropy generation, the ecological benefit and thermo-economic objectives. The criteria were utilized by Sahin and Kodal [19], Kodal and Sahin [20] on the endoreversible and irreversible heat engines. Newly, the optimum operation conditions of an endoreversible solar-driven heat engine with different heat transfer laws in the thermal couplings, operating at maximum ecological function conditions are studied by Barranco-Jimenez et al. [21]. The thermoeconomics of an endoreversible heat engine model under maximum ecological conditions is investigated by Barranco-Jimenez and Angulo-Brown [22], considering different heat transfer laws in a Novikov model. In thermoeconomic analysis of power plant models, an objective function is determined in terms of the cost involved in the performance of the power plant and a characteristic function such as power output [23] and ecological function. The thermoeconomic

**Nomenclature**

|           |                                                                      |                     |                                                                      |
|-----------|----------------------------------------------------------------------|---------------------|----------------------------------------------------------------------|
| $A_{res}$ | absorber (recuperate) area                                           | $t$                 | cyclic period, s                                                     |
| $A_{app}$ | aperture area                                                        | $x$                 | temperature ratio of the Stirling engine                             |
| $C$       | concentration ratio                                                  | $X$                 | vector of decision variables                                         |
| $d$       | deviation index                                                      |                     |                                                                      |
| $d_{i+}$  | deviation or distance of $i$ th solution from the ideal solution     | <b>Greek letter</b> |                                                                      |
| $d_{i-}$  | deviation or distance of $i$ th solution from the non-ideal solution | $\lambda$           | ratio of volume during the regenerative processes (volumetric ratio) |
| $F$       | dimensionless objective function                                     | $\eta$              | thermal efficiency                                                   |
| $h$       | heat transfer coefficient, $W K^{-4}$ or $W m^{-2} K^{-1}$           | $\varepsilon$       | emissivity factor                                                    |
| $I$       | direct solar flux intensity, $W m^{-2}$                              | $\delta$            | Stefan–Boltzman coefficient                                          |
| $i$       | $i$ th objective                                                     |                     |                                                                      |
| $j$       | $j$ th solution                                                      | <b>Subscripts</b>   |                                                                      |
| $n$       | mole number of the working fluid, mol                                | $H$                 | absorber (heater)                                                    |
| $p$       | dimensionless output power                                           | $f$                 | convection at the high temperature side heat exchanger               |
| $Q$       | heat transfer, J                                                     | $L$                 | heat sink                                                            |
| $R$       | universal gas constant, $J mol^{-1} K^{-1}$                          | $k$                 | convection at the low temperature side heat exchanger                |
| $S$       | Entropy                                                              | $m$                 | entire solar dish Stirling system                                    |
| $T$       | temperature, K                                                       | $t$                 | Stirling engine                                                      |
| $W$       | work, J                                                              | $0$                 | Ambient condition, optics                                            |
| $V$       | Volume                                                               | $4 - 1$             | process states                                                       |

nomics of a Novikov power plant model is scrutinized by De Vos [24] in terms of the maximization of an objective function described as the quotient of the power output and the performing costs of the plant.

Investigation of Solar Collector Design Parameters Effect onto Solar Stirling Engine Efficiency was studied by Ahmadi and Hosseinzade [25]. Intelligent approach to estimate solar Stirling heat engine power by implementation of evolutionary algorithm was extended by Ahmadi et al. [26]. Tlili investigated effects of regenerating effectiveness and heat capacitance rate of external fluids in a heat source/ sink at maximum power and efficiency [27]. A mathematical model for the overall thermal efficiency of solar powered high temperature differential dish-Stirling engine with finite heat transfer, irreversibility of regenerator, optimizing the absorber temperature and corresponding thermal efficiency was evolved by Yaqi [28].

Sharma et al. [29] studied the optimization of a solar-powered Stirling heat engine with the finite-time thermodynamics.

The multi-objective optimization obstacle is a very tough purpose to be obtained which needs to satisfy a number of distinct and even contradictory objectives. At first, evolutionary algorithms (EA) were developed and applied in the 18th century to stochastically solve problems of this generic class [30]. A logical alternative to a multi-objective problem is to interrogate a collection of solutions, which satisfies the objectives at a decent level without being overcome by any other solutions [31]. Generally, Multi-objective optimization problems illustrate a large number of solutions so-called Pareto frontier, whose evaluated vectors represent the best trade-offs result. In this field, multi-objective optimization of different thermodynamic and energy systems have been inquired by researchers [32–39].

In the present investigation, multi-objective optimization is conducted to achieve the optimal dimensionless power and thermal efficiency of a dish-Stirling system with respect to thermoeconomic parameter. Five decision variables such as temperature ratio, irreversibility factor, heat source temperature, hot working fluid temperature, ratio of heat transfer areas (heat sink to heat source) are considered. The presented analysis is great at designing and suitable design parameters can be obtained based on the analysis. The paper is organized as follows: In Section 2, we present the

thermodynamic and thermoeconomic description of the system, we apply the first and second laws of Thermodynamics to construct the objective functions, in Section 3 we describe the multi-objective optimization using evolutionary algorithms, in Section 4, we describe our decision-making methods implemented in our work, in Section 5 we present our main results and finally, in Section 6, we present our conclusions.

## 2. Thermodynamic analysis of the system

In solar-dish Stirling systems, mirrors of the parabolic shaped concentrator focuses the sun light on the focal point of the concentrator where the hot end of the Stirling engine is located. Therefore,

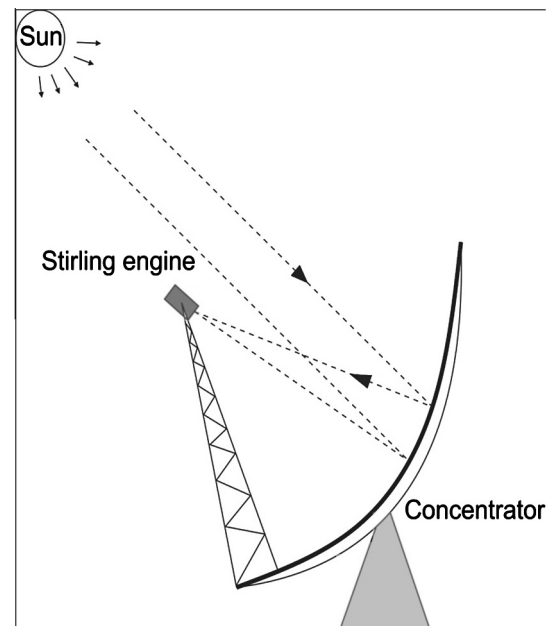


Fig. 1. Schematic diagram of dish-Stirling system [28].

the solar energy with a relatively high temperature is transferred to the hot side of heat exchanger of the Stirling engine. Fig. 1 illustrates a schematic for a solar-dish Stirling engine connected to a solar dish concentrator. The solar-dish is equipped with a sun tracker which tracks the sun in order to have maximum solar energy transfer to the engine when sun moves during the day. Hence, the solar energy is absorbed and transferred to the working fluid in the hot space of engine.

As shown in Fig. 2, the Stirling cycle consists of four processes. Process 1–2 is an isothermal process, in which the compressing working fluid at constant temperature,  $T_c$ , rejects the heat to a heat sink at constant temperature,  $T_L$ . Then the working fluid crosses over the regenerator and is warmed up to  $T_h$  in an isochoric process 2–3. In process 3–4, the working fluid expands at a constant temperature,  $T_h$ , and receives the heat from the heat source at a constant temperature  $T_H$ . Last process (4–1), is an isochoric cooling process, where the regenerator absorbs the heat from the working fluid.

Actual useful heat gain of the dish collector, considering conduction, convection and radiation losses is given by [26–29] as follows:

$$q_u = IA_{app}\eta_0 - A_{rec} \left[ h(T_H - T_0) + \varepsilon\delta(T_H^4 - T_0^4) \right] \quad (1)$$

where  $I$  is the direct solar flux intensity,  $A_{app}$  is the collector aperture area,  $\eta_0 = 0.85$  is the collector optical efficiency,  $A_{rec}$  is the absorber area,  $h$  is conduction/convection heat transfer coefficient,  $T_{H_{ave}}$  is the (absorber temperature) heat source temperature,  $T_0$  is the ambient temperature,  $\varepsilon$  is emissivity factor of the collector and  $\delta$  is the Stefan's constant.

Thermal efficiency  $\eta_s$  of the dish collector is obtained as [26–29]:

$$\eta_s = \frac{q_u}{IA_{app}} = \eta_0 - \frac{1}{IC} \left[ h(T_H - T_0) + \varepsilon\delta(T_H^4 - T_0^4) \right] \quad (2)$$

where  $C$  is the collector concentrating ratio.

### 2.1. The amounts of heat released by the heat source and absorbed by the heat sink

The heat released between heat source and working fluid ( $Q_h$ ), the heat absorbed between the working fluid and the heat sink ( $Q_c$ ) is obtained as follows [28]:

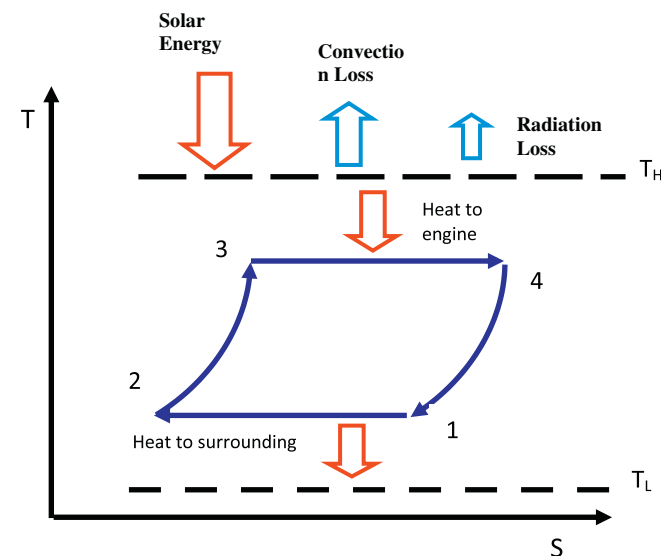


Fig. 2. Heat flows involved in a Stirling engine with solar collector.

$$Q_h = [h_h A_H (T_H - T_h)] t_h \quad (3)$$

$$Q_c = h_c A_L (T_c - T_L) t_l \quad (4)$$

where  $t_l$  and  $t_h$  are the times required for engine heat rejection and engine heat accumulation, respectively.  $T_c$  and  $T_h$  are the temperature of the working fluid in the isothermal heat rejection and addition process, respectively.  $A_H$  denotes heat transfer area for heat exchanger of hot side and  $A_L$  represents heat transfer area for the heat exchanger of the cold side in Stirling engine.  $h_h$  is high temperature side convection heat transfer coefficient and  $h_c$  is low temperature side convection heat transfer coefficient.

In the isothermal and endothermic process, the entropy term in the thermodynamic relation is:

$$TdS = T \left( \frac{dP}{dT} \right)_v dV \quad (5)$$

The ideal gas equation is:

$$PV = nRT \quad (6)$$

Substituting Eq. (6) into Eq. (5) yields:

$$TdS = \frac{nRT}{V} dV \quad (7)$$

Integrating Eq. (7) over states 3 and 4 yields:

$$\int_3^4 TdS = \int_3^4 \frac{nRT}{V} dV \quad (8)$$

Entropy is defined as:

$$dS = \left( \frac{\delta Q}{T} \right)_{int,rev} \quad (9)$$

Substituting Eq. (9) into Eq. (8) yields:

$$Q_{34} = Q_h = nRT_h \ln \left( \frac{V_4}{V_3} \right) = nRT_h \ln \lambda \quad (10)$$

Let volumetric ratio  $\lambda$  be defined as:

$$\lambda = \frac{V_4}{V_3} = \frac{V_1}{V_2} \quad (11)$$

In each cycle, variation in entropy in the working fluid inside a Stirling engine is obtained by:

$$\Delta S = \oint \frac{\delta Q}{T} + S_{gen} \quad (12)$$

Since entropy is a state parameter it is independent of the integration path. It should be noted that here, since processes are developed at the working fluid level ( $T_h$ ,  $T_c$ ),  $S_{gen}$  is due only to internal irreversibility. Therefore, the net change in entropy between the initial and final states of a complete cycle is identically zero:

$$\Delta S = \oint \frac{\delta Q}{T} + S_{gen} = \frac{Q_h}{T_h} - \frac{Q_c}{T_c} + S_{gen} = 0 \quad (13)$$

From the Second Law of thermodynamics:

$$S_{gen} \geq 0 \quad (14)$$

The Clausius inequality is:

$$\oint \frac{\delta Q}{T} \leq 0 \quad (15)$$

From Eqs. (13)–(15):

$$\frac{Q_h}{T_h} - \frac{Q_c}{T_c} \leq 0 \quad (16)$$

Where the irreversibility parameter can be seen as a measure of the departure from the endoreversible case. Now, let [4,5]:

$$\frac{Q_h}{T_h} = \frac{Q_c}{\phi T_c}, \quad \phi \geq 1 \quad (17)$$

where  $\phi$  is an irreversible property of a Stirling engine, such as thermal resistance, friction, heat loss. Consistent with the second law of thermodynamics, such irreversible factors, which cannot be ignored, bring about an entropy generation. Given an identical amount of heat transfer  $Q_h$ , for an endoreversible heat engine  $\phi=1$ ,  $Q_c$  is expressed as:

$$\frac{Q_h}{T_h} - \frac{Q_c^{rev}}{T_c} = 0 \quad (18)$$

Relating Eqs. (17) and (18) yields:

$$Q_c = \phi Q_c^{rev} \quad (19)$$

In a Stirling engine, the heat that is released to low-temperature thermal storage (cold chamber) in a reversible isothermal process yield:

$$Q_c^{rev} = nRT_c \ln \left( \frac{V_1}{V_2} \right) \quad (20)$$

Back-substituting Eq. (20) into Eq. (19) yields:

$$Q_c = \phi nRT_c \ln \left( \frac{V_1}{V_2} \right) = \phi nRT_c \ln \lambda \quad (21)$$

Equating Eq. (3) with Eq. (10) yields:

$$[h_h A_H (T_H - T_h)] t_h = nRT_h \ln \lambda \quad (22)$$

Time of heat transferred process 3–4 is given by:

$$t_h = \frac{nRT_h \ln \lambda}{h_h A_H (T_H - T_h)} \quad (23)$$

Equating Eq. (4) with Eq. (21) yields:

$$h_c A_L (T_c - T_L) t_l = \phi nRT_c \ln \lambda \quad (24)$$

Time of heat transferred process 1–2 is given by:

$$t_l = \frac{\phi nRT_c \ln \lambda}{h_c A_L (T_c - T_L)} \quad (25)$$

Irreversibility of the finite-rate heat transfer causes the following fact that the time of the regenerative processes is not insignificant in comparison to the two isothermal processes [28]. To compute the time of the regenerative processes, the temperature of the working fluid in the regenerative processes is deemed as a function of time which is discerned by [5,26–29]:

$$\frac{dT}{dt} = \pm M_i \quad (26)$$

where  $M$  is the proportionality constant which is independent of the temperature difference and depends only on the property of the regenerative material, called regenerative time constant and the  $\pm$  sign belong to the heating ( $i=1$ ) and cooling ( $i=2$ ) processes respectively [5,26–29].

$$t_3 = \frac{T_h - T_c}{M_1} \quad (27)$$

$$t_4 = \frac{T_h - T_c}{M_2} \quad (28)$$

The cyclic period

Using Eqs. (22)–(28), we get that the cyclic period  $t$  is:

$$t = \frac{nRT_h \ln \lambda}{h_h A_H (T_H - T_h)} + \frac{\phi nRT_c \ln \lambda}{h_c A_L (T_c - T_L)} + \left( \frac{1}{M_1} + \frac{1}{M_2} \right) (T_h - T_c) \quad (29)$$

## 2.2. The conductive thermal bridging losses from heat source to the heat sink

This value proportional to the temperature difference of the heat source and heat sink and the cycle time is obtained as follows [28]:

$$Q_0 = k_0 (T_H - T_L) t \quad (30)$$

The net heat released from the heat source ( $Q_H$ ) and the net heat absorbed by the heat sink ( $Q_L$ ) are obtained as follows:

$$Q_H = Q_h + Q_0 \quad (31)$$

$$Q_L = Q_c + Q_0 \quad (32)$$

## 2.3. Power and thermal efficiency

Considering the cyclic period of the Stirling engine, the output power, the thermal efficiency of the engine are given by:

$$P' = \frac{W}{t} = \frac{Q_H - Q_L}{t} \quad (33)$$

$$\eta_t = \frac{Q_H - Q_L}{Q_H} \quad (34)$$

Substituting Eqs. (22)–(32) into Eqs. (33) and (34) we have,

$$P' = \frac{T_h - \phi T_c}{\left( \frac{T_h}{h_h A_H (T_H - T_h)} + \frac{\phi T_c}{A_R A_H h_c (T_c - T_L)} + F_1 (T_h - T_c) \right)} \quad (35)$$

$$\eta_t = \frac{T_h - \phi T_c}{T_h + k_0 (T_H - T_L) \left[ \frac{T_h}{h_h A_H (T_H - T_h)} + \frac{\phi T_c}{A_R A_H h_c (T_c - T_L)} + F_1 (T_h - T_c) \right]} \quad (36)$$

where  $F_1 = \frac{1}{nR \ln \lambda} \left( \frac{1}{M_1} + \frac{1}{M_2} \right)$ .

For the sake of convenience, a new parameter  $x = \frac{T_c}{T_h}$  is introduced into Eqs. (35) and (36), then we have:

$$P' = \frac{(1 - \phi x)}{\left( \frac{1}{h_h A_H (T_H - T_h)} + \frac{\phi x}{A_R A_H h_c (x T_h - T_L)} + F_1 (1 - x) \right)} \quad (37)$$

$$\eta_t = \frac{(1 - \phi x)}{T_h + k_0 (T_H - T_L) \left[ \frac{1}{h_h A_H (T_H - T_h)} + \frac{\phi x T_h}{A_R A_H h_c (x T_h - T_L)} + F_1 (1 - x) \right]} \quad (38)$$

where the  $A_R$  represents heat transfer area ratio  $\left( \frac{A_L}{A_H} \right)$ .

The thermal efficiency of the dish-Stirling system is product of the thermal efficiency of the collector and the thermal efficiency of the Stirling engine [28,29]:

$$\eta_m = \eta_t \eta_s \quad (39)$$

## 2.4. Thermo-economic model

In the present inquiry, the objective function is determined as the power output per unit investment cost since there is no fuel consumption cost in a solar-driven heat engine [19,20,23]. For optimizing power output per unit total cost, the objective function is revealed as follows:

$$F' = \frac{P'}{C_{ai}} \quad (40)$$

where  $C_{ai}$  refers to the annual investment cost. The investment cost of the plant is assumed to be proportional to the size of the plant. The size of the plant can be taken proportional to the total heat transfer area. Thus, the annual investment cost of the system can be given as [18–23]

$$C_{ai} = (a A_H + b A_L) \quad (41)$$

Where the investment cost proportionality coefficients for hot- and cold-sides are (a) and (b), respectively, equal to the capital recovery factor times investment cost per unit heat transfer area, and the dimension is  $\text{ncu}/(\text{yr m}^2)$ .

Substituting Eqs. (37) and (41) into Eq. (40), we obtain

$$F' = \frac{(1 - \phi x)}{a \left( \frac{1 + \left(\frac{1-f}{f}\right) A_R}{h_h(T_H - T_h)} + \frac{\phi x \left(1 + \left(\frac{1-f}{f}\right) A_R\right)}{A_R h_c(x T_h - T_L)} + F_1 A_H (1 - x) \left(1 + \left(\frac{1-f}{f}\right) A_R\right) \right)} \quad (42)$$

where  $f$  is relative investment cost parameter of the hot-side heat exchanger and defined as [18–23]

$$f = \frac{a}{a + b} \quad (43)$$

By using Eqs. (37), (39) and (42) one can obtain the dimensionless thermo-economic objective function  $F = \frac{a F'}{h_h T_L}$ , dimensionless power output  $P = \frac{P'}{h_h A_H T_L}$  and thermal efficiency of the dish-Stirling system  $\eta_m$ , respectively, as

$$F = \frac{(1 - \phi x)}{T_L \left( \frac{1 + \left(\frac{1-f}{f}\right) A_R}{(T_H - T_h)} + \frac{\phi h_h x \left(1 + \left(\frac{1-f}{f}\right) A_R\right)}{A_R h_c(x T_h - T_L)} + F_1 h_h A_H (1 - x) \left(1 + \left(\frac{1-f}{f}\right) A_R\right) \right)} \quad (44)$$

$$P = \frac{(1 - \phi x)}{T_L \left( \frac{1}{(T_H - T_h)} + \frac{\phi h_h x}{A_R h_c(x T_h - T_L)} + F_1 h_h A_H (1 - x) \right)} \quad (45)$$

$$\eta_m = \left\{ \eta_0 - \frac{1}{IC} \left[ h(T_H - T_0) + \varepsilon \delta (T_H^4 - T_0^4) \right] \right\} \times \frac{(1 - \phi x)}{T_h + k_0(T_H - T_L) \left[ \frac{1}{h_h A_H (T_H - T_h)} + \frac{\phi x T_h}{A_R A_H h_h (x T_h - T_L)} + F_1 (1 - x) \right]} \quad (46)$$

### 3. Multi-objective optimization with evolutionary algorithms

#### 3.1. General concepts of the multi-objective optimization

Imagine a decision-maker that would like to optimize objectives which are non-commensurable. The decision-maker does not know the priority of the objectives. Moreover, all objectives are of the minimization type. For a minimization kind of objectives, it can be changed to the maximization type by multiplying negative one. The procedure of a minimization multi-objective decision problem with objectives is represented as follows:

Offer an non-dimensional decision variable vector  $x = \{x_1, \dots, x_n\}$  in the solution area  $X$ . Detect a vector  $x^*$  which minimizes a collection of  $K$  objective functions  $f(x) = \{f_1(x^*), \dots, f_k(x^*)\}$ . The solution space  $X$  is limited by a number of constraints, such as  $g_j(x^*) = b_j$  for  $j = 1, \dots, m$ , and bounds on the decision variables [31]. Totally, it is not possible to minimize all the  $K$  objective functions at once with one solution vector  $X$  which means in real-world issues, objectives at issues disagree with each other. Thus, optimizing  $X$  with concerning a uni-objective mostly propels to unreasonable outcome with respect to the other objectives. Thus a novel implication, so-called the “Pareto optimum solution”, is employed in multi-objective optimization quandaries. A practical solution  $X$  is denominated “Pareto optimal” if there is no other plausible solution  $Y$  that prevails solution  $X$ . Therefore, a feasible solution  $Y$  is entitled to overcome another conceivable solution  $X$ , if and only if,  $f_i(Y) \leq f_i(X)$  for  $i = 1, \dots, k$  and  $f_j(Y) < f_j(X)$  for at least one objective function which denotes that a possible vector  $X$  is denominated

Pareto optimal if there is no other possible solution  $Y$  that would decrease some objective functions without concluding a simultaneous increase in at least one other objective function. The collection of all plausible non-dominated solutions in  $X$  is leaded to the “Pareto optimal set”, and for a certain Pareto optimal collection, the corresponding objective function values in the objective space are denominated the “Pareto optimal frontier” [31].

Fig. 3 illustrates the objective functions space for an optimization problem with two objective functions  $f_1$  and  $f_2$ . The feasible area is insight and infeasible area is outisight of the curve. Each point in the feasible area denotes a solution. It is obvious that the values of functions  $f_1$  and  $f_2$  for point  $M$  are lower than the corresponding values of point  $J$ . Therefore point  $M$  dominates point  $J$  or point  $M$  is better than point  $J$ . In the same way points  $L$  and  $N$  dominate  $M$ . But points  $I$  and  $K$  neither overcome  $M$  nor are overcome by  $M$ . Thus, only three points that are placed in the left-down parts of  $M$  would overcome  $M$ . There is not any point in the feasible space placed in the left-down part of point  $R$ . Thus  $R$  is not overcome by the other points in the feasible space and hence  $R$  is a Pareto optimal. This is true for points  $P, A, R, E, T, O$  and all of the other points placed at the bold curve denoted as “The Pareto Optimal Frontier” in Fig. 3. In the feasible space, the minimum value of  $f_1$  is for  $P$  and the minimum value of  $f_2$  is for point  $O$ . Thus  $P$  and  $O$  are the solutions for one objective optimization issues whose objective functions are  $f_1$  and  $f_2$ , respectively. The other points on the Pareto frontier are also optimal, but a decision-making procedure is needed to find which of them should be selected.

#### 3.2. Optimization via EA

In the present investigation, the Pareto frontier is achieved using the Genetic Algorithm (GA) which is identified as a field of evolutionary algorithm. Genetic Algorithms were evolved by John Holland in the 1960s to import the mechanisms of natural adaptation into computer algorithms and numerical optimization [33]. They are applied as a computer simulation in a way that a population of abstract representations (called chromosomes or the genotype of the genome) of candidate solutions (called individuals, creatures, or pheno-types) to an optimization problem develops toward better solutions. It commences from a population of randomly produced exclusives and occurs in generations. In every generation, the fitness of each individual is measured; multiple individuals are stochastically opted from the current population, and altered to form a new population. The new group is employed

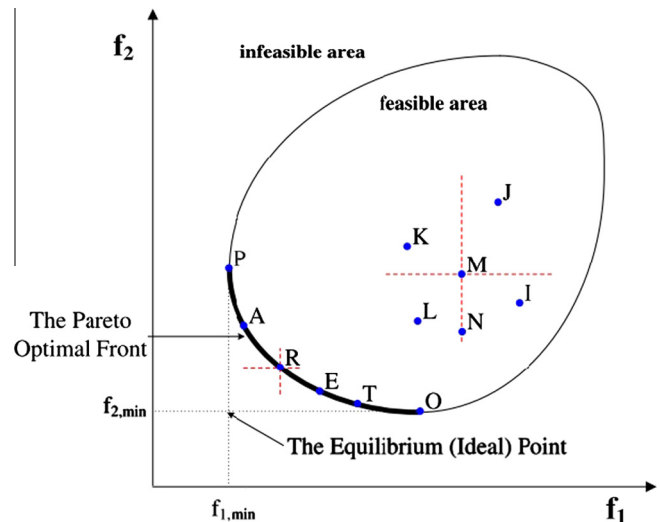


Fig. 3. Schematic of the objectives space.



in the next iteration. Generally, the algorithm finishes when either a maximum number of generations have been produced, or a satisfactory fitness level has been obtained for the population. If the algorithm has ended owing to a maximum number of generations, a satisfactory solution might have been reached or might not have been accomplished. In Genetic Algorithms, a candidate solution to a problem is named a chromosome, and the evolutionary viability of each chromosome is denoted by a fitness function. This method can be applied to optimize non-linear problems [31,34].

Multi-objective evolutionary algorithms (MOEAs) have been evolved in the recent decades by a lot of tests on complex mathematical problems. On real-world it has shown that they can omit the obstacles of classical methods [31,34]. The structure of the MOEA exploited in this investigation is depicted in Fig. 4 [33]. The real values of decision variables are applied instead of their binary coded.

### 3.3. Objective functions, decision variables and constraints

Three objective functions for this study are the dimensionless thermo-economic objective function ( $F$ ), the dimensionless power output ( $P$ ) and the thermal efficiency of dish-Stirling system ( $\eta_m$ ) denoted by Eqs. (44)–(46), respectively.

In this paper five decision variables have been considered as follow:

- $\phi$  Internal irreversibility parameter
- $A_R$  Heat transfer area ratio ( $\frac{A_L}{A_H}$ )
- $x$  Temperature ratio ( $\frac{T_c}{T_h}$ )
- $T_H$  Temperature of heat source
- $T_h$  Temperature of the working fluid in the high temperature isothermal process

The objective functions with respect to following constraints have been solved:

$$0.45 \leq x \leq 0.7 \quad (47)$$

$$1100 \leq T_H \leq 1400 \text{ K} \quad (48)$$

$$\phi \geq 1 \quad (49)$$

$$850 \leq T_h \leq 1000 \text{ K} \quad (50)$$

$$0.25 \leq A_R \leq 10 \quad (51)$$

## 4. Decision-making in the multi-objective optimization

In multi-objective issues, a procedure of decision-making to opt the optimal solution from existing results is needed. There are lots of procedures for decision-making which can be applied to pick the final solution from the Pareto frontier that is achieved through

optimization. At first, dimensions and scales of objectives space must be unified due to the possibility of different dimensions of objectives in multi-objective optimization issues. In turn, objectives vectors must be non-dimensionalized before the decision-making procedure. Some methods of non-dimensionalization including linear, Euclidian, and fuzzy can be employed in decision making.

### • Linear non-dimensionalization

Consider the matrix of objectives at various points of the Pareto frontier is denoted by  $F_{ij}$  where  $i$  is index for each point on the Pareto frontier and  $j$  is the index for each objective in objectives space. Therefore a non-dimensionalized objective,  $F_{ij}^n$ , is defined as,

$$F_{ij}^n = \frac{F_{ij}}{\max(F_{ij})} \quad \text{For maximizing objective} \quad (52a)$$

$$F_{ij}^n = \frac{F_{ij}}{\max(1/F_{ij})} \quad \text{For minimizing objective} \quad (52b)$$

### • Euclidian non-dimensionalization

In this method, a non-dimensionalized objective,  $F_{ij}^n$ , is defined as,

$$F_{ij}^n = \frac{F_{ij}}{\sqrt{\sum_{i=1}^m (F_{ij})^2}} \quad \text{For minimizing and maximizing objectives} \quad (53)$$

### • Fuzzy non-dimensionalization

In this method, a non-dimensionalized objective,  $F_{ij}^n$ , is defined as,

$$F_{ij}^n = \frac{F_{ij} - \min(F_{ij})}{\max(F_{ij}) - \min(F_{ij})} \quad \text{For maximizing objectives} \quad (54a)$$

$$F_{ij}^n = \frac{\max(F_{ij}) - F_{ij}}{\max(F_{ij}) - \min(F_{ij})} \quad \text{For minimizing objectives} \quad (54b)$$

In the present investigation, some well-known and common kinds of decision-making procedures including the fuzzy Bellman-Zadeh, LINMAP and TOPSIS methods are applied in parallel and the final optimal solution has been opted based on real-world engineering experience and criteria among solutions which are recommended by the mentioned ways. The Bellman-Zadeh procedure implements the fuzzy non-dimensionalization while the other ways (LINMAP and TOPSIS) use Euclidian non-dimensionalization. The algorithms are introduced in the following.

### 4.1. Bellman-Zadeh decision-making method

When using the Bellman-Zadeh approach, each  $F_j(X)$  is replaced by a fuzzy objective function or a fuzzy set,

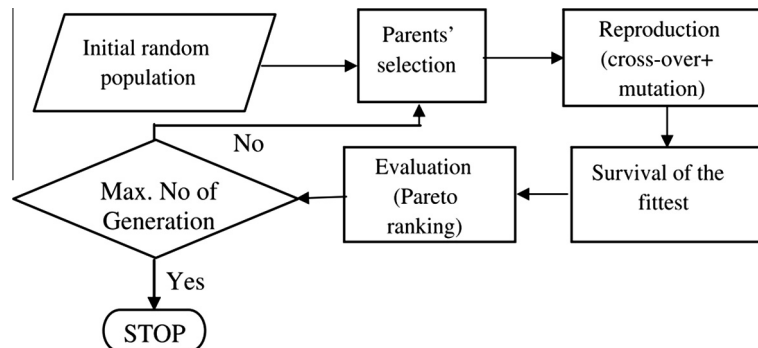


Fig. 4. Scheme for the multi-objective evolutionary algorithm used in the present study.

$$A_j = \{X, \mu_{A_j}(X)\} \quad X \in L, \quad j = 1, 2, \dots, k \quad (55)$$

where  $\mu_{A_j}(X)$  is a membership function of  $A_j$  [40].

A final decision is defined by the Bellman and Zadeh model as the intersection of all fuzzy criteria and constraints and is represented by its membership function. A fuzzy solution  $D$  with setting up the fuzzy sets (Eq. (56)) is turned out as a result of the intersection  $D = \cap_{j=1}^k A_j$  with a membership function,

$$\mu_D(X) = \cap_{j=1}^k \mu_{A_j}(X) = \min_{j=1, \dots, k} \mu_{A_j}(x), \quad X \in L \quad (56)$$

Using Eq. (54a), it is possible to obtain the solution proving the maximum degree as follows:

$$\max \mu_D(X) = \max_{X \in L} \min_{j=1, \dots, k} \mu_{A_j}(x) \quad (57)$$

$$X^0 = \arg \max_{X \in L} \min_{j=1, \dots, k} \mu_{A_j}(x) \quad (58)$$

To obtain Eq. (57), it is necessary to build membership functions  $\mu_{A_j}(X); j = 1, \dots, k$ , reflecting a degree of achieving “own” optimal by the corresponding  $F_j(X), X \in L; j = 1, \dots, k$ . This is satisfied by the use of the membership functions [41]. The membership function of objectives and constraints, linear or nonlinear, can be chosen depending on the context of problem. One of possible fuzzy convolution schemes is presented below [42].

Initial approximation for  $X$ -vector is chosen. Maximum (minimum) values for each criterion  $F_j(X)$  are established via scalar maximization (minimization). Results are denoted as “ideal” points  $\{X_j^0; j = 1, \dots, m\}$ .

The matrix table  $\{T\}$ , where the diagonal elements are “ideal” points, is defined as follows:

$$\{T\} = \begin{bmatrix} F_1(X_1^0) & F_2(X_1^0) & \dots & F_n(X_1^0) \\ F_1(X_2^0) & F_2(X_2^0) & \dots & F_n(X_2^0) \\ \vdots & \vdots & \ddots & \vdots \\ F_1(X_n^0) & F_2(X_n^0) & \dots & F_n(X_n^0) \end{bmatrix} \quad (59)$$

Maximum and minimum bounds for the criteria are defined:

$$F_i^{\min} = \min_j F_j(X_j^0), \quad i = 1, \dots, n \quad (60a)$$

$$F_i^{\max} = \max_j F_j(X_j^0), \quad i = 1, \dots, n \quad (60b)$$

The membership functions are assumed for all fuzzy goals as follows.

For minimized objective functions:

$$\mu_{F_i}(X) = \begin{cases} 0 & \text{if } F_i(x) > F_i^{\max} \\ \frac{F_i^{\max} - F_i}{F_i^{\max} - F_i^{\min}} & \text{if } F_i^{\min} < F_i \leq F_i^{\max} \\ 1 & \text{if } F_i(x) \leq F_i^{\min} \end{cases} \quad (61a)$$

For maximized objective functions:

$$\mu_{F_i}(X) = \begin{cases} 1 & \text{if } F_i(x) > F_i^{\max} \\ \frac{F_i - F_i^{\min}}{F_i^{\max} - F_i^{\min}} & \text{if } F_i^{\min} < F_i \leq F_i^{\max} \\ 0 & \text{if } F_i(x) \leq F_i^{\min} \end{cases} \quad (61b)$$

Fuzzy constraints are formulated:

$$G_j(X) \leq G_j^{\max} + d_j, \quad j = 1, 2, \dots, k \quad (62)$$

where  $d_j$  is a subjective parameter that denotes a distance of admissible displacement for the bound  $G_j^{\max}$  of the  $j$ th constraint. Corresponding membership functions are defined in following manner:

$$\mu_{G_i}(X) = \begin{cases} 0 & \text{if } G_i(x) > G_i^{\max} \\ 1 - \frac{G_i(x) - G_j^{\max}}{d_j} & \text{if } G_i^{\max} < G_i(x) \leq G_i^{\max} + d_j \\ 1 & \text{if } G_i(x) \leq G_i^{\max} \end{cases} \quad (63)$$

A final decision is determined as the intersection of all fuzzy criteria and constraints represented by its membership functions. This problem is reduced to the standard nonlinear programming problems: to find such values of  $X$  and  $k$  that maximize  $k$  subject to

$$\begin{aligned} \lambda &\leq \mu_{F_i}, \quad i = 1, 2, \dots, n \\ \lambda &\leq \mu_{G_j}, \quad j = 1, 2, \dots, k \end{aligned} \quad (64)$$

The solution of the multi-criteria problem discloses the meaning of the optimality operator and depends on the decision-makers experience and problem understanding.

#### 4.2. LINMAP decision-making method

The point that every objective is optimized irrespective to satisfaction of the others is an ideal point on the Pareto frontier. It is obvious that having every objective in the optimal condition is not possible for the multi-objective optimization and it can achieve in a single-objective optimization problem. In the result, the ideal point is not placed on the Pareto frontier. In the LINMAP procedure, after Euclidian non-dimensionalization of all existing objectives, the distance of every solution on the Pareto frontier from the ideal point marked by  $d_{i+}$  is defined as:

$$d_{i+} = \sqrt{\sum_{j=1}^n F_{ij} - F_j^{\text{ideal}}} \quad (65)$$

where denotes the number of objective while stand for each solution on the Pareto frontier ( $i = 1, 2, \dots, m$ ). In Eq. (65),  $F_j^{\text{ideal}}$  is the ideal value for  $j$ th objective obtained in a single-objective optimization. In LINMAP method, the solution with minimum distance from ideal point is selected as a final desired optimal solution, hence,  $i$  index for a final solution,  $i_{\text{final}}$  is,

$$i_{\text{final}} = i \in \min(d_{i+}); \quad i = 1, 2, \dots, m \quad (66)$$

#### 4.3. TOPSIS decision-making method

In this method beside the ideal point a non-ideal point is defined. The non-ideal point is the ordinate in objectives space in which each objective has its worst value. Therefore, beside the solution distance from ideal point,  $d_{i+}$ , the solution distance from non-ideal point denoted by  $d_{i-}$  is implemented as a criterion for selection of the final solution. Hence,

$$d_{i-} = \sqrt{\sum (F_{ij} - F_j^{\text{non-ideal}})^2} \quad (67)$$

In continuing the TOPSIS method a parameter is defined as follows,

$$Cl_i = \frac{d_{i-}}{d_{i+} + d_{i-}} \quad (68)$$

In TOPSIS method a solution with minimum  $Cl_i$  is selected as a desired final solution, therefore, if  $i_{\text{final}}$  is an index of the final selected solution, we have,

$$i_{\text{final}} = i \in \max(Cl_i); \quad i = 1, 2, \dots, m \quad (69)$$

### 5. Results and discussion

The dimensionless thermo-economic objective function ( $F$ ), dimensionless output power ( $P$ ) and thermal efficiency of the solar-dish Stirling System ( $\eta_m$ ) are maximized simultaneously using

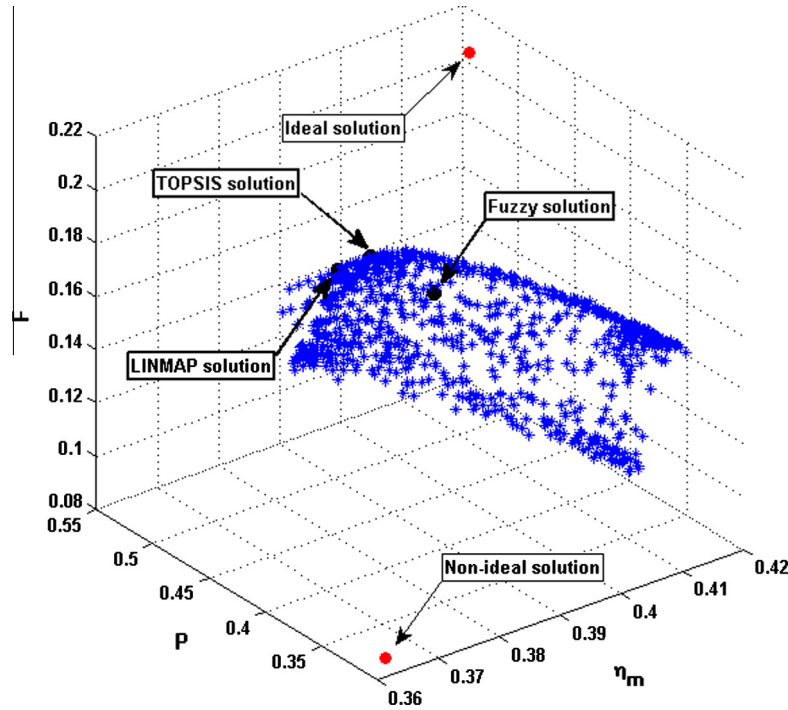
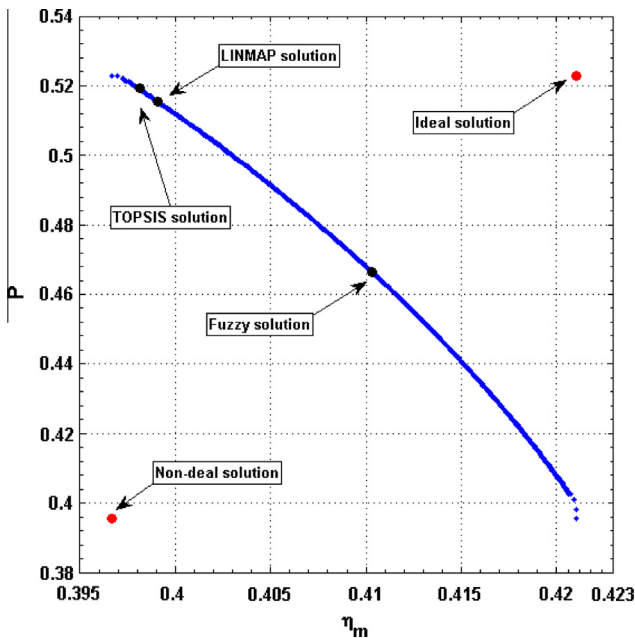
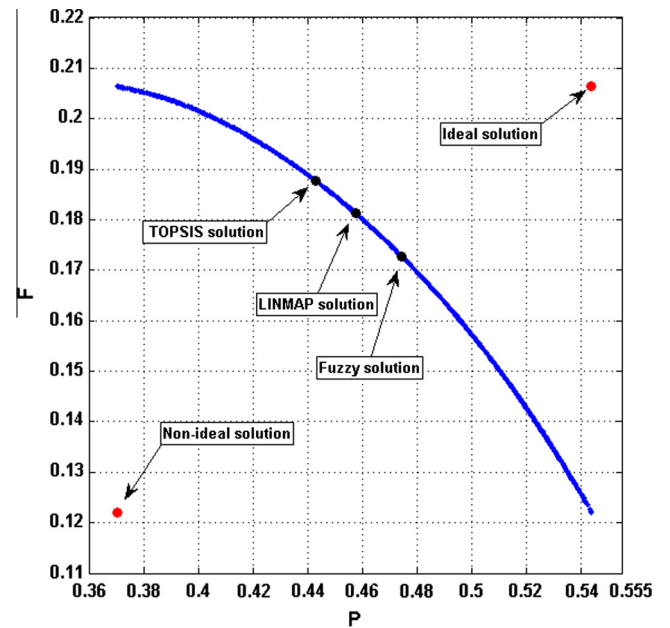


Fig. 5. Pareto optimal frontier in objectives' space.

Fig. 6. Pareto optimal frontier in objectives' space ( $\eta_m - P$ ).Fig. 7. Pareto optimal frontier in objectives' space ( $P - F$ ).

the multi-objective optimizing method which works based on the NSGA-II algorithm. In this regard, optimization is performed with objective functions that are expressed by Eqs. (44)–(46) and constraints which are expressed with Eqs. (47)–(51). Design parameters (decision variables) of optimization are the temperature of the working fluid in the high temperature isothermal process ( $T_h$ ), temperature ratio ( $x$ ), heat transfer area ratio ( $A_R$ ), internal irreversibility parameter ( $\phi$ ) and the absorber temperature ( $T_H$ ).

In order to have consistency with previous works, specifications of the solar-dish Stirling engine are considered as follows [28]:

$$\begin{aligned} h_f &= h_k = 200 \text{ W K}^{-1} \text{ m}^{-2}, f = 0.7, C = 1300, \\ \delta &= 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}, T_L = 320 \text{ K}, T_0 = 300 \text{ K}, \\ h &= 20 \text{ W m}^{-2} \text{ K}^{-1}, I = 1000 \text{ W m}^{-2}, \\ \left( \frac{1}{M_1} + \frac{1}{M_2} \right) &= 2 \times 10^{-5} \text{ s K}^{-1}, R = 4.3 \text{ J mol}^{-1} \text{ K}^{-1}, \\ n &= 1 \text{ mol}, \lambda = 2, \varepsilon = 0.9, k_0 = 2.5 \text{ W K}^{-1}, \eta_0 = 0.85 \end{aligned}$$

Fig. 5 shows optimal results for  $P$ ,  $\eta_m$  and  $F$  objective functions. Optimal results based on three decision making methods are shown and ideal and non-ideal consequences are indicated.



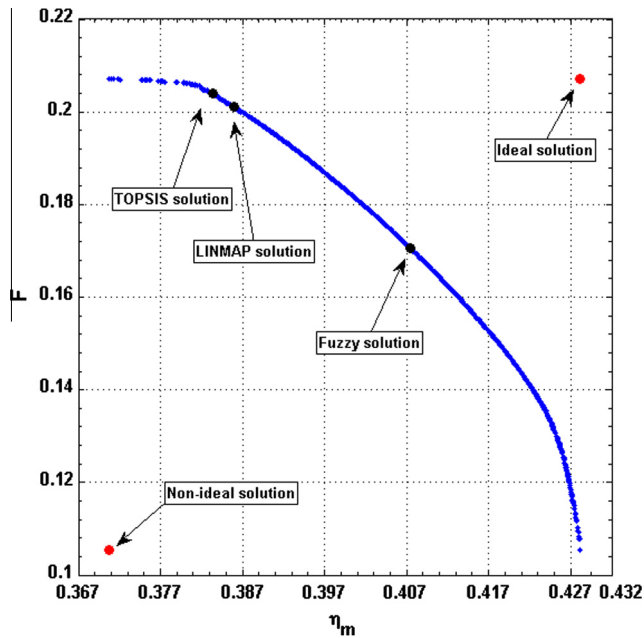


Fig. 8. Pareto optimal frontier in objectives' space ( $F - \eta_m$ ).

Optimal solutions population in  $0.45 \leq P \leq 0.55$ ,  $0.38 \leq \eta_m \leq 0.41$  and  $0.15 \leq F \leq 0.19$  can be seen in Fig. 5. Therefore, optimal

answers are probably in this region and as it is mentioned, the results of Fuzzy Bellman-zade, Linmap and Topsis decision-making methods are in this region. Ideal solution is in coordination (0.42, 0.55, 0.205) which are  $F$ ,  $P$  and  $\eta_m$ , respectively. Non-ideal solution is in coordination (0.365, 0.345, 0.08). The chosen results through Topsis and Linmap methods are the closest to ideal solution. With respect to 0.35–40% thermal efficiency of dish-Stirling system, optimal solutions region is appropriate.

Optimal results for  $P$  and  $\eta_m$  objective functions are illustrated in Fig. 6 and obtained temperatures through decision making methods are displayed.

Optimal solution population for two objective optimization ( $P - \eta_m$ ) are in  $0.392 \leq P \leq 0.524$  and  $0.397 \leq \eta_m \leq 0.421$ . The ideal and non-ideal solutions are in coordination (0.421, 0.524) and (0.397, 0.392), respectively.

Fig. 7 demonstrates optimal results for  $P$  and  $F$  objective functions which are in  $0.37 \leq P \leq 0.545$  and  $0.122 \leq F \leq 0.205$ . Non-linear relationship between  $P$  and  $F$  can be seen. Ideal and non-ideal solution are (0.545, 0.205) and (0.37, 0.122), respectively.

Optimal results for  $\eta_m$  and  $F$  objective functions are represented in Fig. 8.

The results of three-objective optimization based on Fuzzy, LINMAP and TOPSIS decision making methods are shown in Table 1a. In this table, results for  $F$ ,  $P$  and  $\eta_m$  objective functions based on five decision variables are categorized. Temperature ratio ( $x$ ) is obtained between 0.45 and 0.46 which is in acceptable range due to previous studies [16,17]. The achieved values for  $\phi$  are close to 1.  $\phi = 1$  represents Carnot reversible engine. Table 1a demonstrates

**Table 1a**  
Decision making of multi-objective optimal solutions.

| Optimal solution | Decision variables |          |          |          |          | Objectives |          |          |
|------------------|--------------------|----------|----------|----------|----------|------------|----------|----------|
|                  | $x$                | $T_H$    | $\phi$   | $T_h$    | $A_R$    | $\eta_m$   | $P$      | $F$      |
| TOPSIS           | 0.45466            | 1371.415 | 1.002271 | 920.6022 | 3.175778 | 0.384027   | 0.436108 | 0.184709 |
| LINMAP           | 0.459212           | 1371.466 | 1.001516 | 919.8346 | 3.671255 | 0.381995   | 0.454229 | 0.176510 |
| Fuzzy            | 0.451457           | 1329.38  | 1.000984 | 913.8662 | 4.34813  | 0.396756   | 0.449328 | 0.156916 |

**Table 1b**  
Decision making of multi-objective optimal solutions ( $\eta_m - P$ ).

| Optimal solution | Decision variables |          |          |          |          | Objectives |          |
|------------------|--------------------|----------|----------|----------|----------|------------|----------|
|                  | $x$                | $T_H$    | $\phi$   | $T_h$    | $A_R$    | $\eta_m$   | $P$      |
| TOPSIS           | 0.45004            | 1340.77  | 1.000153 | 896.0217 | 8.012228 | 0.398154   | 0.519160 |
| LINMAP           | 0.450119           | 1335.903 | 1.000144 | 896.0208 | 8.016084 | 0.399064   | 0.515551 |
| Fuzzy            | 0.450056           | 1273.835 | 1.000111 | 896.0406 | 8.005058 | 0.410305   | 0.466470 |

**Table 1c**  
Decision making of multi-objective optimal solutions ( $P - F$ ).

| Optimal solution | Decision variables |          |          |          |          | Objectives |          |
|------------------|--------------------|----------|----------|----------|----------|------------|----------|
|                  | $x$                | $T_H$    | $\phi$   | $T_h$    | $A_R$    | $P$        | $F$      |
| TOPSIS           | 0.465289           | 1382.388 | 1.000085 | 912.5242 | 3.169516 | 0.442871   | 0.187788 |
| LINMAP           | 0.461091           | 1382.428 | 1.000114 | 911.3326 | 3.555935 | 0.457653   | 0.181323 |
| Fuzzy            | 0.455245           | 1382.369 | 1.000028 | 911.1519 | 4.072863 | 0.474479   | 0.172820 |

**Table 1d**  
Decision making of multi-objective optimal solutions ( $\eta_m - F$ ).

| Optimal solution | Decision variables |         |          |          |          | Objective functions |          |
|------------------|--------------------|---------|----------|----------|----------|---------------------|----------|
|                  | $x$                | $T_H$   | $\phi$   | $T_h$    | $A_R$    | $\eta_m$            | $F$      |
| TOPSIS           | 0.450132           | 1373.37 | 1.000107 | 948.427  | 1.798122 | 0.383051            | 0.204405 |
| LINMAP           | 0.450049           | 1357.56 | 1.000065 | 947.5719 | 1.807338 | 0.386469            | 0.200713 |
| Fuzzy            | 0.450135           | 1262.96 | 1.000066 | 918.7098 | 1.835071 | 0.403693            | 0.176783 |

**Table 2**

Error analysis based on the mean absolute percent error (MAPE) method.

| Decision making method | LINMAP   |   |   | TOPSIS   |   |   | Fuzzy    |    |    |
|------------------------|----------|---|---|----------|---|---|----------|----|----|
|                        | $\eta_m$ | P | F | $\eta_m$ | P | F | $\eta_m$ | P  | F  |
| Max error %            | 1        | 5 | 6 | 1        | 4 | 2 | 1        | 10 | 10 |
| Average error %        | 0        | 1 | 3 | 1        | 2 | 1 | 1        | 5  | 5  |

that the efficiency of dish-Stirling system is between 38.4% and 39.67% which is acceptable because of previous works [28,29].

The results of two-objective optimization ( $P$  and  $\eta_m$ ) based on three mentioned decision making methods are represented in Table 1b. The consequences of two-objective optimization for ( $F, P$ ) and ( $\eta_m, F$ ) objective functions are illustrated in Tables 1c and 1d, respectively.

Mean absolute percentage error was carried out in error analysis of implemented methods. To assess this goal, 30 runs of each approach were performed to get final solution by bellman-zadeh, linmap and topsis decision making methods. First row of Table 2 demonstrate maximum absolute percentage error (MAAE) of three decision making methods. Moreover, second row of Table 2 report mean absolute percentage error (MAPE) of mentioned methods. Finally it worth bearing in mind that, all of the above procedure encoded in the MATLAB® software.

## 6. Conclusions

Throughout current research work enormous efforts have been made to figure out optimum condition of the three addressed objective function such as dimensionless output power, thermal efficiency of dish-Stirling system and dimensionless thermo-economic parameter while through developed multi-objective optimization approach five different variables like as heat source temperature, hot working fluid temperature, temperature ratio and irreversible property were considered as decision variables. In addition, the effects of the referred decision variables on each objective-function like as thermal efficiency of dish-Stirling system were carried out while different decision making approaches implemented. Due to obtained solutions from different decision making approaches, fuzzy decision making has higher maximum error percent than other two addressed decision making approaches such as LINMAP and TOPSIS approaches. This fact could be illustrated by independency of the fuzzy decision making approach on ideal solution and non-ideal solution. In other hand, based on maximum error percent, obtained routs of TOPSIS approach has superiority than other approaches like as LINMAP. Optimal results which are closer to ideal result are chosen. Stirling engine thermal efficiency is between 40% and 45% and dish-Stirling thermal efficiency is between 35% and 40% in turn, the results of the presented thermo-economic model for dish-Stirling implementing NSGA-II algorithm are acceptable. It should be noted that, Starting with the scheme for computation of the Performances (Efficiency and Power) for Solar Stirling Engines developed on Thermodynamics with Finite Speed [6–12] future papers which will utilize the methods described in the present communication will obtain results closer to the reality. The authors intend to do such a progress in the near future.

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